The average efficiency factor is calculated from the harmonic mean of the canonical efficiency factors. The canonical efficiency factors are obtained from the eigenvalues divided by the treatment replication number, i.e.

,

where denotes the canonical efficiency factor corresponding to the th eigenvalue, , and denotes the treatment replication. Alternatively, the canonical efficiency factors can be obtained directly from the eigenvalues of the information matrix divided *r*, which is given by.

if all treatments are equally replicated, or by

,

otherwise, where is a diagonal matrix with th diagonal element equal to (John and Williams, 1987) and is its inverse. Typically, for an experiment with only one treatment factor, the treatment information matrix of th stratum is given by

where denotes the treatment design matrix and denotes the projector for the th stratum. The treatment design matrix is a binary matrix whose (*l*,*m*)th element corresponds to the *l*th observation on an experimental unit assigned treatment *m*, where *m* denotes a factorial treatment combination. Thus, for a factorial experiment in which factor *F*1 and *F*2 have *v*1and *v*2 levels, respectively, and in which each of the *v* = *v*1*v*2 factorial treatment combinations is replicated *r* times, *X* will have dimensions *rv* × *v*. For experiments with two treatment factors, in addition to *X*, treatment contrast matrices are needed to decompose the information associated with the main effects of, and interaction between, treatment factors, denoted by , and , respectively.

Now, consider a two-factor factorial experiment in which the first treatment factor, *F*1, has *v*1 = 2 levels and the second treatment factor, *F*2, has *v*2 = 3 levels, the dimensions of the contrast matrices are . Thus, the treatment information matrix for the first treatment factor (i.e. the *F*1 main effect) in the th stratum is given by

Since the dimension of this information matrix is , the dimension of also has to be . Note that for *F*1 *r* = n/2. Thus, *r* is multiplied by 2/6 because the information matrix is written with respect to the factorial treatment combinations. Hence, the treatment replication number for treatment factor 1 in a matrix can be written as,

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Note that rows and columns of this matrix correspond to the combination of treatment factor 1 and 2, i.e. , where and denote treatment factor 1 and 2 respectively. The replication matrix for treatment factor 1 can then be re-written as

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The advantage of this it allows the design that contains unequal treatment replication because the replication is based on each treatment combination separately. Hence, the canonical efficiency factors should still sum to one which is what it supposed to be.